Geometrical Nonlinear Interaction Model of Thin Insulating Glass Units

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Abstract

The use of glass for exterior façade has been becoming increasingly popular, as it offers more light to the inhabitants and gives an aesthetic view to the building itself. Due to the increase in glass demand, sophisticated engineering analysis comes into question in the design for optimization purposes. It is a well-known fact that, accounting for the finite deformations in the analysis of glass-unit plate leads to quite economical designs. Although, it is rather simple to perform nonlinear analysis on a single-plate glass-unit with Finite Element Method (FEM), it is not the case for insulating glass-panes. In this study, we performed the analysis of insulating glass units for rectangular geometry with distinct size, thickness and loading cases, while considering the force-exchange between the multiple panes in a simplified model. Mechanical model of the system is created as two rectangular plates with air as the insulating cavity in between. Volume changes of the cavity and the corresponding pressures are calculated in an iterative procedure in which plate deflections are calculated by nonlinear FEM and internal pressure changes are computed according to the Boyle's Law. The results are compared with the non-interacting case and with the relevant design codes. Geometrical nonlinear interaction of the plates cannot be underestimated as seen from the results. It is found that, the model can be used for geometrical nonlinear interaction of thin insulating glass units to build durable and structures.

Keywords: *Insulating, Modeling, Nonlinear, Plate*

1 Introduction

Plates are widely used structural members reflecting the behavior of the structures which have one dimension smaller than other two dimensions. Insulating glass units are a combination of plates to construct more durable structures. Modern structures such as high-rise buildings or airports generally are built with using insulating glass units. Two or much more rectangular glass plates placed parallel to each other to generate insulating glass units.

There are very specific studies in the literature to reflect and to analyze the structural behavior of insulating plates. Chou et al. made laboratory studies on glass panels and obtained significant investigations on the structural-mechanics behavior of insulating glass units and wrote reports about this topic (Chou et al., 1986). Load sharing in insulating glass units was studied as considered linear and nonlinear plate deflection analysis (Wörner et al., 1993). Finite element models to analyze the nonlinear behavior of glass panels with using a shell element were performed (Kwok et al., 1995). Seismic behavior of nonstructural window glass panels was investigated (Sucuoğlu and Vallabhan, 1995). They also performed a study about the behavior of window glass panels during earthquake (Sucuoğlu and Vallabhan, 1997). Other researchers were produced a method for predicting approximate lateral deflections in thin glass plates (Xenidis et al., 2015). In another study, a verification formula for structural glass under combined variable loads was presented (Franco and Carfagni, 2015). A comparative study of methods to determine load sharing of insulating glass units for environmental loads was performed. (Morse and Norville, 2016).

In this paper, a nonlinear model on the analyzing of thin insulating plates presented effectively by using the finite element method. Structural performance of the insulating plates is investigated. In addition, some remarkable results are obtained about the structural design parameters of the insulating glass units.

2 Finite Element Formulation of Insulating Units

There is a complicated interaction between plates and air because of the change in volume inside the insulating plates. The change of volume brings about the pressure change. There are some assumptions made to create a model reflecting the insulating glass unit behavior. Elastic and isotropic material are used in the models. There is any temperature change both inside and outside of the insulating glass units. The initial pressure in the insulating glass units is assumed to be equal to the external pressure which is equal to the atmospheric pressure 101.3 kPa. The following is the Classical Plate Theory also available in (Reddy, 2004) which we implemented in a finite element code to perform the load-deflection analysis of the glass panes. Figure 1 depicts the displacement and rotation fields along with the selected discretization scheme for the finite element analysis.

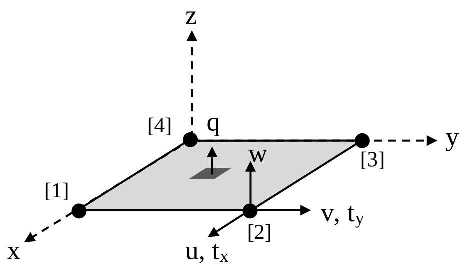


Figure 1: The selected 4-node shell element.

Let u, v, and w be the displacement fields in the local x, y and z directions, respectively. The constitutive model of the element can be summarized as;

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^0 = u_{,x} + 0.5w_{,x}^2 \\ \varepsilon_{yy}^0 = v_{,y} + 0.5w_{,y}^2 \\ \gamma_{xy}^0 = u_{,y} + v_{,x} + 0.5w_{,x}w_{,y} \end{Bmatrix} \Rightarrow \quad \mathbf{N} = [\mathrm{A}]\mathbf{e}^0,$$
$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^1 = -w_{,xx} \\ \varepsilon_{yy}^1 = -w_{,yy} \\ \gamma_{xy}^1 = -2w_{,xy} \end{Bmatrix} \Rightarrow \quad \mathbf{M} = [\mathrm{D}]\mathbf{e}^1 \tag{1}$$

where A_{ij} and D_{ij} are extensional and bending stiffness coefficients, respectively, which are defined in terms of the plane-stress case the medium with the constitutive model in Eqn. (2).

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}, \quad A_{ij} = hQ_{ij}, \quad D_{ij} = \frac{h^3}{12}Q_{ij},$$
$$Q_{11} = \frac{E}{1-v^2}, \quad Q_{22} = \frac{E}{1-v^2}, \quad Q_{12} = \frac{vE}{1-v^2}, \quad Q_{66} = \frac{E}{2(1+v)} \tag{2}$$

The finite element equations can be obtained by using Virtual Work Theorem;

$$\iint_A \left(N_x \frac{\partial \bar{u}}{\partial x} + N_{xy} (\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x}) + N_y \frac{\partial \bar{v}}{\partial y} \right) \mathrm{d}A + \\ \iint_A \left(\frac{\partial \bar{w}}{\partial x} (\frac{\partial w}{\partial x} N_x + \frac{\partial w}{\partial y} N_{xy}) + \frac{\partial \bar{w}}{\partial y} (\frac{\partial w}{\partial x} N_{xy} + \frac{\partial w}{\partial y} N_y) \right) \mathrm{d}A + \\ -\iint_A \left(M_x \frac{\partial^2 \bar{w}}{\partial x^2} + 2M_{xy} \frac{\partial^2 \bar{w}}{\partial x \partial y} + M_y \frac{\partial^2 \bar{w}}{\partial y^2} \right) \mathrm{d}A = \iint_A \bar{w} q(x,y)\, \mathrm{d}A \tag{3}$$

where the terms with over-bar are the variations of the displacement fields. As the Figure 2 depicts the selected directional convention for the inner and outer panes of the insulating glass unit,

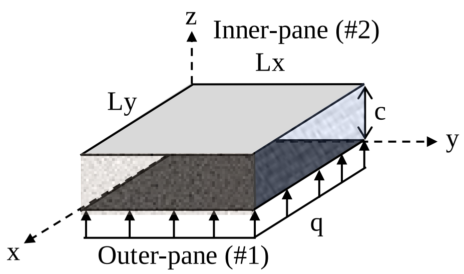


Figure 2: Two-layer insulating glass unit

Eqn. (3) can be expressed in a compact form for the individual panes as,

$$\mathbf{k}_1(\mathbf{w}_1) = \mathbf{q} - \mathbf{p} \\ \mathbf{k}_2(\mathbf{w}_2) = \mathbf{p} \tag{4}$$

here $\mathbf{p}$ is the external load vector due to the pressure change p in the cavity. Using the Boyle-Mariotte Law, one can easily verify that, Eqn. (5) must hold simultaneously with Eqn. (4),

$$k_3(p) = p(V_c + \Delta V_c) + p_{env} \Delta V_c = 0 \tag{5}$$

where V_c and p_{env} are the initial volume of the cavity and the environment pressure before the loading respectively and ΔV_c is the volume change of the cavity. We obtained a stable solution by implementing the iterative procedure for Eqn. (5) with the form,

$$\left. \frac{dk_3}{dp} \right|_{p=p_i} \Delta p = \left. \left((V_c + \Delta V_c) + (p + p_{env}) \frac{d\Delta V_c}{dp} \right) \right|_{p=p_i} \Delta p = -k_3(p_i) \tag{6}$$

In order to incorporate the Eqn. (6) with Eqn. (4), the term $d\Delta V_c / dp$ is calculated by solving Eqn. (4) twice, first for $\mathbf{p} \to \mathbf{p}_i$ and the second for $\mathbf{p} \to \mathbf{p}_i + \delta\mathbf{p}$, so the derivative of the volume change with respect to the cavity pressure is calculated from the deformed geometry of both solutions. It is worth to mention that; this procedure is observed to be numerically very stable in terms of fast convergence to a proper solution.

3 Numerical Results and Discussion

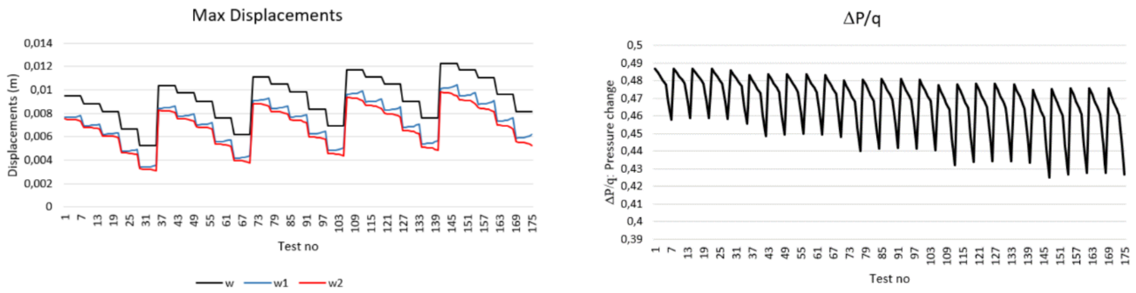
Two insulating glass units with different square dimensions are investigated as the first example. Table 1 shows the selected dimensions. For the material properties Young's Modulus and Poisson’s ratio is taken as E=70 GPa,

v=0,24. A mesh study was performed before obtaining the structural results. Thus, 2x2 mesh distribution is accepted enough for the analysis on the insulating glass panes. Analysis are performed in two distinct cases in terms of equal-pane insulating glass units for different dimensions and loading conditions (Mesh: 2x2) as shown in Table 1.

Table 1. Equal-pane insulating glass units test properties (Mesh: 2x2).

Case	Lx(m)	Ly (m)	h (mm)	q (kPa)	c (cm)
Pane(2x2)	2	2	4, 5, 6, 8,10	0.6, 0.8, 1.0, 1.2, 1.4	1.2, 1.4, 1.6, 1.8, 2.0, 3.0, 4.0
Pane(3x3)	3	3			

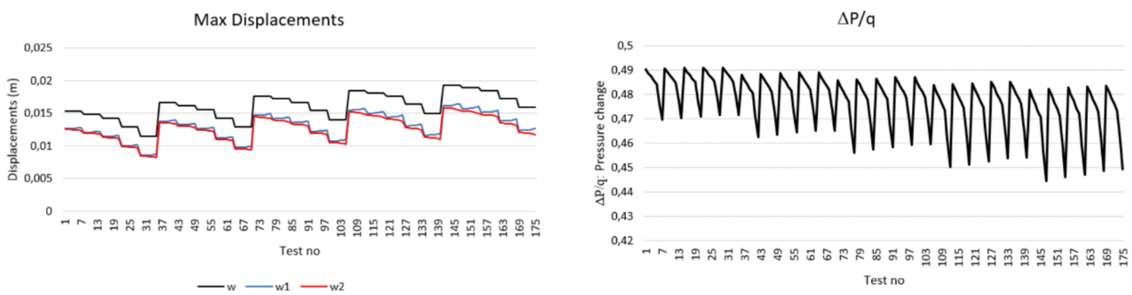
A parameter α =ΔP/q is determined to analyze the results clearly. Here ΔP is the change in pressure value obtained by the finite element analysis. The finite element analyzes results of the equal-pane insulating glass with 2x2 dimensions are shown in Figure 3. The wind load values q is changed with every 35 tests according to Table 1. Every 7 tests in these 35 tests results, the thickness of the equal panes h is also changed. In addition, the value of c which is determined as the distance between the pane is changed with all the tests as seen in the Table 1. The changing of the thickness of the panes causes a stepwise seeming as seen in Figure 3a. The pressure changing between the panes is shown in Figure 3b. The pressure changing occurs approximately between the values of 0.49 to 0.43.



(a) (b)

Figure 3. For the 2x2 insulating glass units with equal thickness, (a) Deflections of panes, (b) Pressure change between the panes

The finite element analyzes results of the equal-pane insulating glass with 3x3 dimensions are shown in Figure 4. The wind load values q is changed with every 35 tests according to Table 1. Every 7 tests in these 35 tests results, the thickness of the equal panes h is also changed. In addition, the value of c which is determined as the distance between the pane is changed with all the tests as seen in the Table 1. The changing of the thickness of the panes causes a stepwise seeming as seen in Figure 4a. The pressure changing between the panes is shown in Figure 4b. The pressure changing occurs approximately between the values of 0.49 to 0.44. It can be reported from these results, the changing of the plate dimensions from 2x2 to 3x3 does not affect the pressure chance dramatically for insulating glass units.



(a) (b)

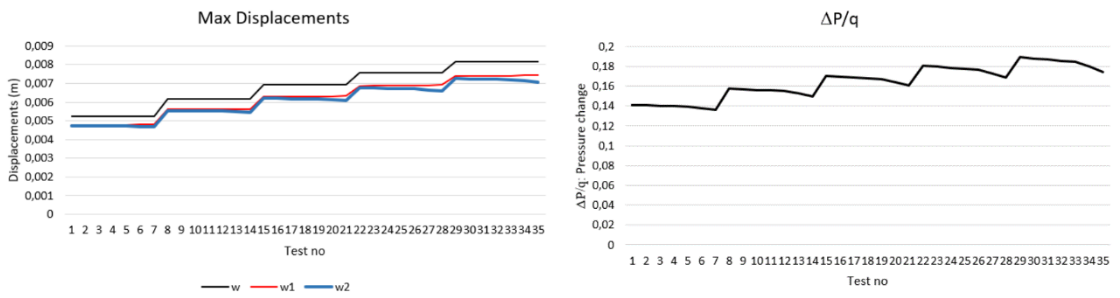
Figure 4. a) For the 3x3 insulating glass units with equal thickness, (a) Deflections of panes (b) Pressure change between the panes

Different panes (thickness changes) insulating glass units for different dimensions and loading conditions (Mesh: 2x2) as shown in table 2. The dimensions of the panes are chosen 2x2 (m) and 3x3(m) for both two cases. The thicknesses of the insulating glass are determined as h1 (outer side) and h2 (inner side) of the insulating system.

Table 2. Different panes (thickness changes) insulating glass units test properties (Mesh: 2x2).

Case	Lx(m)	Ly (m)	h1 (m)	h2 (m)	q (kPa)	c (cm)
Pane(2x2)	2	2	4 or 10	10 or 4	0.6, 0.8, 1.0, 1.2, 1.4	1.2, 1.4, 1.6, 1.8, 2.0, 3.0, 4.0
Pane(3x3)	3	3				

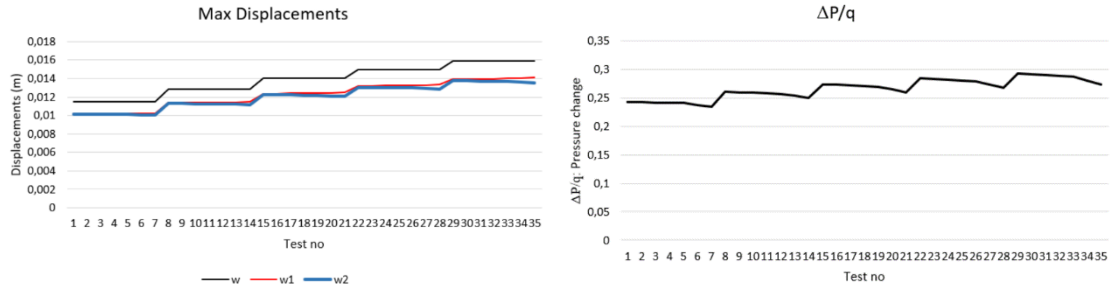
The finite element analyzes results of the different panes (thickness changes) with 2x2 dimension insulating glass units are indicated in Figure 5. The wind load values q is changed with every 7 tests according to Table 2. The thicknesses of the insulating glass are determined as h1 (outer side) =4 mm and h2 (inner side) =10 mm of the insulating system. In addition, the value of c which is determined as the distance between the pane is changed with all the tests as seen in the Table 2. The magnitude of the pressure causes a stepwise seeming as seen in Figure 5a. The pressure changing between the panes is shown in Figure 5b. The pressure changing occurs approximately between the values of 0.14 to 0.19. It can be reported from these results; the changing of the plate gaps significantly effects the pressure chance.



(a) (b)

Figure 5. For the 2x2 insulating glass units with distinct thickness, (a) Deflections of panes, (b) Pressure change between the panes

The finite element analyzes results of the different panes (thickness changes) with 3x3 dimension insulating glass units are indicated in Figure 6. The wind load values q is changed with every 7 tests according to Table 2. The thicknesses of the insulating glass are determined as h1 (outer side) =4 mm and h2 (inner side) =10 mm of the insulating system. In addition, the value of c which is determined as the distance between the pane is changed with all the tests as seen in the Table 2. The magnitude of the pressure causes a stepwise seeming as seen in Figure 5a. The pressure changing between the panes is shown in Figure 5b. The pressure changing occurs approximately between the values of 0.25 to 0.29. It can be reported from these results; the changing of the plate gaps significantly effects the pressure chance and dimensions of the plates produce some pressure changing for insulating glass units.



(a) (b)

Figure 6. For the 3x3 insulating glass units with distinct thickness, (a) Deflections of panes, (b) Pressure change between the panes

4 Conclusion

Geometrical nonlinear interaction model of thin insulating glass units is modeled and analyzed in this study. A finite element model is generated with using the plate element which reflecting the large deflection behavior. A load sharing formulation is proposed to investigate the nonlinear interaction of plates. The results are obtained for the pressure value, the gaps between the plates, and thickness of the plates. It is seen that, a load sharing mechanism between the plates are produced after imposed the wind loads. The same thickness panes behave similarly. However, if the panes are used which have distinct pane thickness, the sharing of the wind load is changed according to thicknesses of the panes. The expressions presented in this study about the geometrical nonlinear interaction of the insulating plates may be used for analyzing of the advanced structural behavior of thin insulating glass units.

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